

# Steady-state Error

The deviation of the output of control system from desired response during steady state is known as **steady state error**. It is represented as  $e_{ss}$ . We can find steady state error using the final value theorem as follows.

Where,

$E(s)$  is the Laplace transform of the error signal,  $e(t)$

Let us discuss how to find steady state errors for unity feedback and non-unity feedback control systems one by one.

## **Steady State Errors for Unity Feedback Systems**

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$$

$$R(s) \longrightarrow r(t)$$

$$C(s) \longrightarrow c(t)$$

T.F.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$C(s) = \frac{R(s) \cdot G(s)}{1 + G(s)}$$

o/p of the summing point,

$$E(s) = R(s) - C(s)$$

$$E(s) = R(s) - \frac{R(s)G(s)}{1 + G(s)}$$

$$E(s) = \frac{R(s) + \cancel{R(s)G(s)} - \cancel{R(s)G(s)}}{1 + G(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G(s)}$$

Consider the following block diagram of closed loop control system, which is having unity negative feedback.

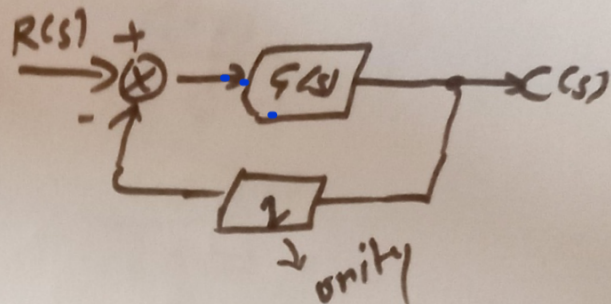
$$e_{ss} = \frac{1}{1+K_p}, \quad \frac{1}{K_v}, \quad \frac{1}{K_a}$$

$$K_p = \lim_{s \rightarrow 0} G(s)$$

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

$$K_a = \lim_{s \rightarrow 0} s^2G(s)$$

Position →  
Velocity →  
acceleration →



Q. f/p signal

$$Y(t) = \left( 5 + 2t + \frac{t^2}{2} \right) u(t)$$

f/p →  $Y_1(t) = 5U(t)$       $e_{ss1} = \frac{5}{1+K_p} = 0$

Error constant →  $K_p = \lim_{s \rightarrow 0} G(s) = \infty$       $e_{ss2} = \frac{2}{K_v} = 0$

Steady state error →  $e_{ss1} = \frac{5}{1+K_p} = 0$       $e_{ss3} = \frac{1}{K_a} = 1$

$$e_{ss} = 0 + 0 + 1 = 1$$

↓  
(unity)

# **Steady State Errors for Non-Unity Feedback Systems**